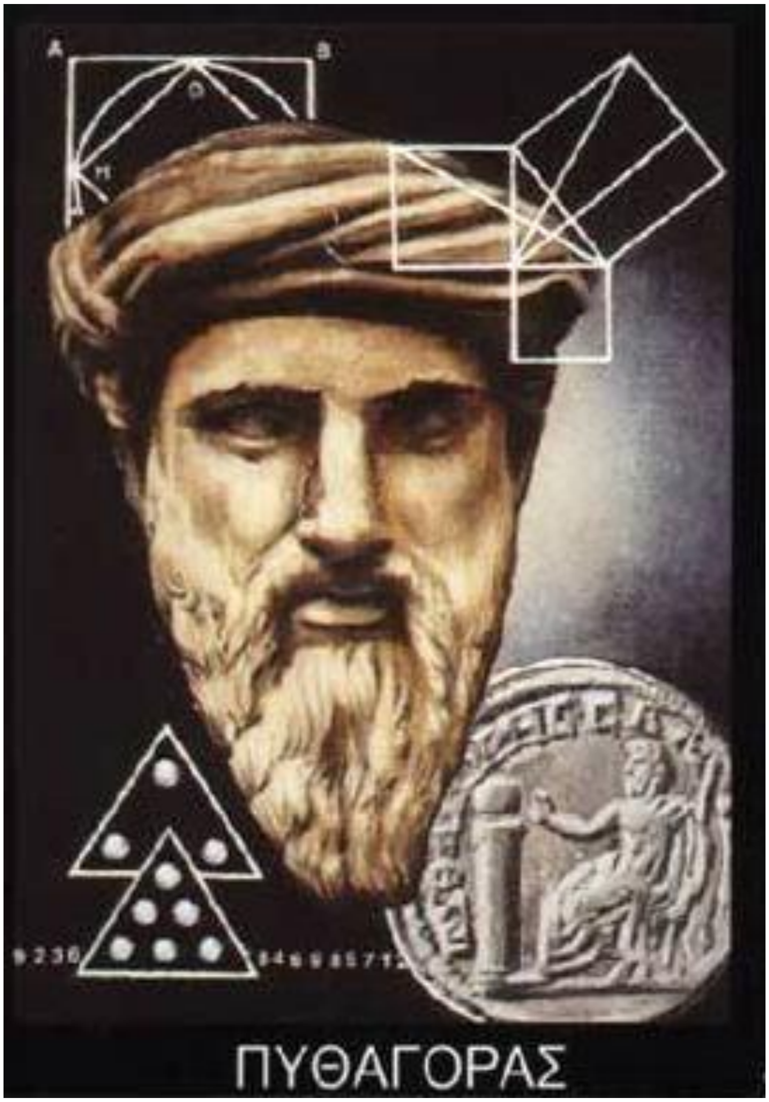
Genius Magazine



Volume Three

Genius Magazine: Volume Three

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‘All we see around us can be characterized by Mathematics.’’

Pythagoras

Prologue

Obelisk Press of Vancouver is proud to publish the third edition of ***Genius Magazine*** which serves to feature the work of unique thinkers. ***Genius Magazine*** welcomes submissions on an annual basis.

The Theme for the third edition of ***Genius Magazine*** is Pythagoras.

Pythagoras is a mysterious figure from ancient times. The reason why we know very little about him is that he managed to antagonize his fellow citizens which meant he was pushed out of his society. He decided to form a secret society and studiy *Mathematicus* … He is the inventor of the term *Mathematicus* as well as *Philosophia*.

We owe a great debt to Pythagoras. In one far reaching statement he said all we see around us can be characterized by Mathematics. For those of us who are mathematicians or physicists we find his statement to be very compelling.

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Pythagoras

Much has been written about the ancient philosopher and mathematician Pythagoras, however, we known few facts about his life. Over the passage of time he has become somewhat of a mythical figure.

Pythagoras of Samos (582-500 B.C.) is best known for the geometric theorem bearing his name. There is sufficient anecdotal evidence to support the view that he may not have been the first to realize the relationship between the sides of a right triangle.

What we do know for certain is that Pythagoras believed that every physical object and process can be represented and understood through numbers. We can ultimately trace the beginnings of mathematics and the physical sciences to Pythagoras.

The Historian of Mathematics and M.I.T. mathematics professor, E. T. Bell in his book *The Magic of Numbers* describes Pythagoras in these words,

> “If one man more than any other is to be credited with starting the mathematical and physical sciences on their course from antiquity to the present it is Pythagoras. And if ‘*Western Civilization*’ means the technology and commerce of recurrent industrial revolutions resulting from the application of experiment and mathematics to the physical world, Pythagoras was its prime mover.”

Chapter Ten of *The Magic of Numbers* has been included as Appendix Three.

Excerpts from E. T. Bell's book *The Magic of Numbers* has been uploaded to archive.org.

Mathematicus …

There are some things we take for granted without even thinking about them. For each of us there are is a personal list of such things. Since *mathematicus* is my middle name, it is a list of things mathematical.

Yes, math is a four letter word! … like exam, quiz and test … but so are the words love and true and hope. Well don't flip the page just yet.

From time to time I tutor high school mathematics for some extra pocket money. Recently, while tutoring a grade 11 student trigonometry, she asked me some rather well thought out questions about the ancient mathematician Pythagoras: where was he was born, where did he grow up, what he was doing when he discovered the law that bears his name, how did he discover his law of right triangles.

She asked a dozen penetrating questions. I tried to answer her questions the best that I could … but it sort of undermined her confidence in her math tutor, until I had to admit that we know much more about his theorem, than we know about Pythagoras himself.

Her honest questions got me wondering – I decided I would set out in search of Pythagoras.

In Search of Pythagoras

Undoubtedly, the most valuable think I own is my library card and so I started with a long and disappointing search on the shelves of the West Vancouver Memorial Library.

It was disappointing not because of the paltry information that was at hand about Pythagoras himself, but the equally wanting lack of good mathematics books on the shelves.

In frustration I asked one of the librarians why such an important place had so few good mathematics books and the answer she gave me sort of threw me for a spin … "*no one reads math books, people prefer graphic novels and cook books instead*."

One of the better sources at the Library turned out to be one of the vintage Encyclopaedia Britannica –circa before 1900, (the Ninth Edition, which I have transcribed as Appendix One).

It struck me as ironic that one of her librarian colleagues recently presented me with a box of mathematics books, the prize and private collection of her recently deceased father. He had taught math at high school for half of his professional life, being a soldier for the first half, a veteran from World War Two, an artillery man who had served with the Canadian Army.

I held out hope that amongst these prized books I would find some jewels about Pythagoras and his life. Jane, his daughter had given me his prize collection with the understanding that they would find a good home. At the time she offered them to me, I asked why they were not placed in the library's collection for everyone to read and enjoy, and she answered in an atypical and monotone voice. In her I heard the same disappointment … "no one reads math books anymore …". Well, I thought, I do.

Some weeks after, and without hesitation I had accepted Jane's wonderful gift, I found myself opening the treasure chest of books. Having been disappointed in my quest amongst the shelves of the old library for background information about Pythagoras and his work I searched the 35 math books that had come into my possession in the way of her kind gift.

As I flipped through the old, musty and yellowing pages of the books, I could not but help notice that all of them were soft cover and all of them were printed for the most part on newsprint quality paper. These were inexpensive books, not meant to last with now long gone printers, mostly London based. This told me that even fifty years ago … no one read mathematics books.

This special and unique collection of math books were interesting, mind you, and many pages wet my appetite for a return visit and a closer read, but what

I found in the way of additional information about Pythagoras was not much more encouraging than what I happen upon in the few books and reference encyclopaedia at the West Vancouver Memorial Library. Even these speciality mathematics books differed in the chronology and “facts” about his life.

Mind you, they agreed he despised beans, but even then, the reasons were not agreed upon. Besides, what does beans have to do with mathematics?

Well undeterred I continued in my search. I pulled a few of my old and trusted history of mathematics books off my shelves or out of storage and pressed on.

From one reputable source I was reminded that Pythagoras of Samos (582-500 B.C.) is best known for the geometric theorem bearing his name. In another, that there is sufficient anecdotal evidence to support the view that he may not have been the first to realize the relationship between the sides of a right triangle. What’s in a name?

Well in his case a great deal. Before I explain why, let me surprise you that Pythagoras is the father of several words that are revered and cherished in our age, some 2,500 years after his death.

Pythagoras is believed to have minted the words ***Philosopher*** – “*a lover of knowledge*” and ***Mathematics*** – “*that what is learned*”. It is obvious then, if you do not like math, you do not like learning!

Then I discovered his name Pythagoras, wasn't even his real name, but a name either given to himself, or to him by his disciples.

The ancient writer and historian Aristippus of Cyrene in his *Account of Natural Philosophers* wrote that Pythagoras derived his name from the fact he was speaking the truth (in Greek – *agoreuein*) like the God of Delphi (*tou Pythiou*).

Delphi was, in ancient times, where the temple of Apollo was located, the greatest of the Greek Gods. In essence, Pythagoras' name was meant to equate him with the greatest of Greek Gods – Apollo himself.

For all we know, when he was born he was Sam from Samos. It was his learning and his unique achievements, which earned him his revered and timeless recognition. In terms of both mathematics, and to a lesser extent philosophy, Pythagoras has become a sort of living god.

I believe it might be safe to say that more men and woman, boys and girls, know of his name and of the theorem that bears his name than any other person, alive or dead, be it Jesus Christ, or Mohammed, Buddha, or Confucius.

If there every was a self-made man in ancient times, perhaps it was Pythagoras, self-made in the same way that the Renaissance genius Leonardo da Vinci was self made, or someone like Freeman Dyson or Albert Einstein, in our modern age.

Pythagoras was a unique thinker that sought to think beyond his age, and through his work and his mathematics and philosophy live a life as if there was to be a timeless tomorrow.

If you put it in your mind that you can't be a thinker, and enjoy life at the same time, what we know of Pythagoras' life would make a good action thriller. He left Samos to escape the stifling tyranny of Polycrates at a time when Samos was at war with the neighbouring city states. Samos ruled the sea and Polycrates had established Naval supremacy in the Aegean sea.

There was a thriving industry of war, a warfare state as it were, in Samos and it was not safe for a thoughtful and inquisitive man like Pythagoras. If he had stayed in Samos, he would have been arrested, probably emasculated, and then chained into servitude as a rower on one of Polycrates' many man powered man-of-war.

Perhaps following the advice of his mentor, the numberist and logician Thales of Miletus, Pythagoras ventured to Egypt, perhaps to Alexandria, an Alexandria before the Great Library of ancient times was built. A numberist, by the way, is someone who enjoys numbers, either in calculation or in numerology. Pythagoras found a safe haven amongst the scholars and numberists of ancient Egypt. He stayed in Egypt for two decades learning their astronomy, and metaphysics.

Many of the scholars in Egypt at the time, like Pythagoras, were exiles from the despotic city states in the Mediterranean and Middle East. In some sense

Alexandria was to its age, what Switzerland is to ours, a thriving metropolis based on banking, scholarship and trade.

When the Persians invaded and conquered Egypt, Pythagoras, along with scores of the other Egyptian scholars and numberists, were gathered together as war booty and plunder. This plunder was a sort of an ancient version of Operation Paperclip, the gathering of German and Japanese scientists and engineers at the end of the Second World War in 1945.

Pythagoras and his colleagues were forcibly put aboard a Persian ship, displaced against their will to Babylon and forced to teach the Persians everything they knew. As a reminder of the plight of the less fortunate in ancient times of war, the scholars and numberists were rowed north by cuckolded and defeated Persian prisoners pulling at the oars of their man-of-war.

It was in Babylon that Pythagoras would have learning of the richness of Mesopotamian numberism. Having learned all there was about the numberism of the age, Pythagoras, sought out more than just the recipes of calculations – the *Algorithms* – as the Arabs numberists called them.

It was then that Pythagoras made two significant discoveries, that numbers and their properties can be thought of as abstract entities that exist in their own right irrespective of recipes and algorithms, and that every physical object and process can be represented and understood through numbers.

With these two discoveries, we can ultimately trace the beginnings of both mathematics and the physical sciences to Pythagoras, twenty-five centuries ago and some 2,000 years before Galileo Galilei, the purported father of modern science.

We can also trace the schism between Metaphysics and Natural Philosophy to those who respected Pythagoras' ideas and those who didn't. For instance, Aristotle, it seems, despised Pythagoras, and belittled him in word and in print, going out of his way to bury Pythagoras' legacy by Aristotle's own.

Well, I know of no Aristotle Theorem worth remembering, do you? Pythagoras was a self-made man, a mathematical existentialist, unequalled in his age, and for many centuries thereafter.

Anyone who has had the misfortune of rubbing elbows with academics in contemporary times, know that today, two and a half millennia after Pythagoras, sharpest elbows and mendaciousness, still prevail amongst that spoiled, idle and opinionated class.

Pythagoras would escape his captivity, after taking from Mesopotamian his own loot and booty, and would go on to create a following or School of Disciples – the *Pythagoreans* – during the course of which he and his disciples would be respected by some and despised by others.

It is worth noting that Pythagoras would live a life twice as long as that of the average life span of his age. Perhaps he looked after himself rather well.

It is said that he asked his disciples to recite a morning mantra:

"As soon as you wake, in order lay
the actions to be done the coming day."

At the end of their day they were to recite the evening mantra:

"Allow not sleep to close your eyes
Before three times reflecting on
Your actions of the day, what deeds
Done well, what not, what left undone?"

The circumstances surrounding Pythagoras' death is that he may have been murdered by ruffians and thugs, the same way that Hypatia, the librarian of the Great Library of Alexandria, fermented to a fervour by factious ranting of Pythagoras' detractors and enemies.

With Pythagoras' demise, civilization would have to wait twenty centuries before someone of similar unique character would be born, an illegitimate baby of an underage mother in Pisa in the 15th century. He too would be ostracized and have to set out making his own name, the ultimate country boy making it big in the big city – Leonardo da Vinci.

On a more personal level, I have actually seen a 10th century Arabic transcription of Pythagoras' Theorem at the Metropolitan Museum in New

York during a special collections presentation of Arab translations of Euclid Elements on loan from the Hermitage Museum in St. Petersburg.

It was a thrilling moment for me to see such a remarkable manuscript. I was amazed on the crispness of the lines and the extraordinarily well – preserved linen upon which this thousand year old manuscript had been drawn. I couldn’t read the Arabic, but I could understand the math.

Pythagoras' Theorem

Sometime during our time at school we are all introduced to a right triangle – a triangle with one angle equal to π/2 radians (90°). We will have been taught that the longest side of the right triangle is called the hypotenuse and that it is the side of the triangle opposite the angle equal to π/2 radians.

We will have also been introduced to the fact that the internal angles to a triangle add up to π radians (180°) and since there are three internal angles to a triangle, that the right angle is the largest of the three angles.

We are then told that the area of a right triangle is

$$\text{Area} = 1/2\ (\text{base})(\text{height})$$

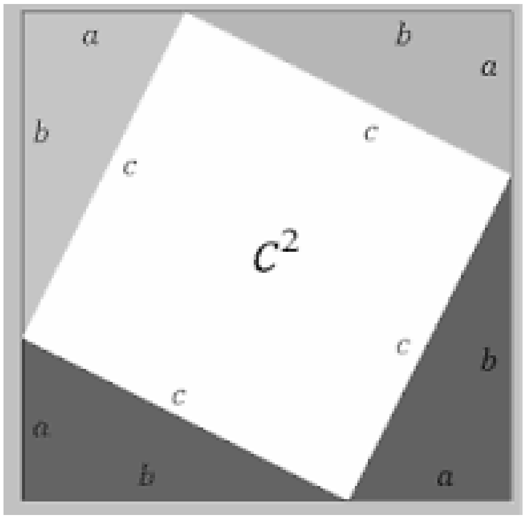
Having accepted these concepts the teacher than says the following, if the hypotenuse has a length c, and the other sides of the triangle have length a and b then the lengths of the three sides are related one to the other two by Pythagoras' Theorem

$$c^2 = a^2 + b^2$$

This is almost universally how Pythagoras' Theorem is taught, and lost forever is the chance to share a little of the magic of mathematics when the proof of Pythagoras' Theorem is not forthcoming.

It turns out the level the theorem is introduced in school, is the level you can prove the theorem. Here is an algorithm to convince yourself of the validity of Pythagoras' Theorem.

Consider a square. Divide each side of the square into measure a and b, where b < a.



From algebra, the total area of the square is

$$\text{Area} = (a + b)(a+b)$$
$$= a^2 + 2ab + b^2$$

Now label each side of the square with the measurements a+b, a+b, a+b, a+b, making sure to keep the same order. Draw a line from where each side is divided at a+b, the previous side where it is divided a+b.

If you have done this correctly you should have now a smaller tilted square inscribed within the larger square and four right triangles of base a and height b. Label the lengths of the side of the smaller tilted square with the letter c (the hypotenuse of the four right triangles).

The total area of the four triangles and the smaller tilted square inscribed within the large square is

$$\text{Area} = c^2 + 4\,[\,1/2\,(ab)]$$
$$= c^2 + 2\,ab$$

From whence we agree that the two measures of the area of the large square must be equal, which means

$$c^2 + 2\,ab = a^2 + 2ab + b^2$$

We cancel out the common term 2ab on both side and arrive at

$$c^2 = a^2 + b^2$$

which is Pythagoras' Theorem.

There is a book kicking around with over 360 derivations of Pythagoras' Theorem. In my view one or two good derivation is enough.

The numbers that come in “Pythagorean Triples”, a, b and c are infinite in number. The first that most students are introduced to are the Triple 3, 4 and 5, namely

$$3^2 + 4^2 = 9 + 16$$
$$= 25 = 5^2$$

How some rather unique Pythagorean Triples can be found is explained in the next section.

Pythagorean Triples

As it happens the theorem that bears Pythagoras’ name may have been known in Babylonia for over a millennia before Pythagoras was even born. A Babylonian clay tablet circa 1600 BC contains fifteen such triples.

The manner in which to find some unique Triples involves basic algebra and is sometimes called Euclid’s formula. Consider and two numbers t and s.

From basic algebra,

$$(t + s)^2 = t^2 + 2ts + s^2$$
$$= t^2 - 2ts + s^2 + 4ts$$
$$= (t - s)^2 + 4ts$$

Now let

$$t = q^2$$

and

$$s = p^2.$$

This means then that (with $p < q$)

$$(q^2 + p^2)^2 = (q^2 - p^2)^2 + 4 q^2 p^2$$
$$= (q^2 - p^2)^2 + (2 q p)^2$$

This is an algorithm for finding a Pythagorean Triple. Chose any two positive integers p and q and you can generate a “Pythagorean Triple” where

$$a = q^2 - p^2$$

$$b = 2 q p$$

and

$$c = q^2 + p^2$$

For example, with q =2 and p = 1 we arrive at the familiar Pythagorean Triple a = 3, b = 4 c = 5. In a Pythagorean Triple all numbers are integers.

With 3 and 2, 4 and 3 … in the same way we find other Triples ; (5, 12, ; 13), (7, 24; 25), (9, 40; 41), (11, 60; 61), … You might have noticed a pattern with this algorithm. You will also notice that you can produce an infinite number of Pythagorean Triples using this algorithm.

There are Pythagorean Triple Questions meant to challenge the reader.

The Cosine Law

Somewhere along the way in high school, after mastering right triangles and Pythagoras' Theorem we are introduced to the Cosine law, namely, if θ is the angle between line a and line b, then

$$c^2 = a^2 + b^2 - 2\,a\,b\cos\theta$$

We are asked to take this equation on faith and memorize it. You recognize immediately that the case where $\theta = \pi/2$ (90°) is the special case of right triangles and leads to Pythagoras' Theorem.

Consider a triangle made of three lines A, B and C. You might draw a triangle to help you visualize the thing. If we consider lines A and B as vectors, that is as order pairs of numbers representing their components, then we can say that $A = (x, y)$ and $B = (v, w)$ (the vector B is longer than A).

The third line C, the hypotenuse of the triangle, is given by the vector $(A - B)$ from which we find

$$C^2 = (A - B)^2$$
$$= A^2 + B^2 - 2\,A \bullet B$$

where $A \bullet B$ is a yet to be determined vector product.

By our definition of the two vectors A and B we see then that multiplying the two “x – components” together and the two “y – components” together

$$A \bullet B = xv + yw$$

Let the longer of the two vectors B define a direction, and measure the angle θ between the two vectors with respect to B being the base of the triangle.

By the manner in which we are measuring the angle,

$$x = A \cos \theta$$

the projection of A onto B, and since $w = 0$, and $v = B$ we find that

$$A \bullet B = (A \cos \theta) B$$

From which we show that

$$C^2 = A^2 + B^2 - 2 A \bullet B$$
$$= A^2 + B^2 - 2 A B \cos \theta$$

which is the cosine law.

Now that wasn’t too hard to show was it?

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Fibonacci Numbers and Pythagorean Triples

Starting with the two terms 0 and 1, the numbers in the Fibonacci Sequence is arrived at by adding together the previous two terms,

$$F(m) = F(m - 1) + F(m - 2)$$

namely

Fibonacci Sequence = 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55,…

There is a brief introduction to the Fibonacci Sequence in Appendix Two. In a rather surprising way, the Fibonacci Sequence is related to Pythagorean Triples.

Take any four consecutive Fibonacci numbers, the product of the outer numbers, as well as twice the product of the inner numbers, and the sum of the square of the inner numbers yield a Pythagorean Triple

Say the four consecutive Fibonacci numbers are say 1,2,3 and 5.

The Product of the Outer Numbers (PON) is

$$PON = 1 \times 5 = 5$$

Twice the Product of the Inner Numbers (PIN) is

$$2 \text{ x PIN} = 2\ (2 \text{ x } 3) = 12$$

The Sum of the Square of the Inner Numbers (SSIN) is

$$\text{SSIN} = 2^2 + 3^2 = 13$$

From which we have

$$(\text{SSIN})^2 = \text{PON}^2 + (2\text{PIN})^2$$

so then, for our example,

$$13^2 = 5^2 + 12^2$$

This is the Pythagorean Triple we derived in a previous section. This remarkable relationship between the Fibonacci Sequence, and the Pythagorean Triples, was discovered by the mathematician Charles Raine.

If this were not strange enough, the Fibonacci Sequence provides a second way as to generate Pythagorean Triples. Starting with 5, every second Fibonacci number is the length of the hypotenuse of a right triangle with integer sides, that is, the largest number in a Pythagorean Triple.

The length of the longer of this right triangle is equal to the sum of the three sides of the preceding triangle in the series, and the shorter side is equal to the difference between the preceding forgotten Fibonacci number and the shorter length of the preceding triangle. Given this added complexity, you can well imagine why the approach is mentioned second.

The first of the triples is 5, 4 and 3. Now skip 8 which is F(6). At 13 which is F(7) we have sides of length

$$13,\ 5 + 4 + 3,\ 8 – 3 = 13,\ 12,\ 5.$$

Skipping 21 = F(8), the next right triangle is at 34 which is F (9) and we have

$$34,\ 13 + 12 + 5,\ 21 – 5 = 34,\ 30,\ 6$$

This algorithm continues indefinitely.

If F(m) represents element m of the Fibonacci Sequence, the triple a_n, b_n and c_n is given by

$$a_n = F(2n – 1)$$

$$b_n = 2\ F(n)\ F(n – 1)$$

$$c_n = F^2(n) – F^2(n – 1)$$

This is mathematical serendipity at its finest, don’t you think?

Fermat Last Theorem

In number theory, Fermat’s Last Theorem derives from the work of Pythagoras and Diophantus.

Diophantus of Alexandria lived in the third century A.D., seven hundred years after Pythagoras. Diophantus is possibly the founder or father of algebra, and the author of thirteen math books titled *Arithmetica*, six of which have survived and seven of which have been lost forever.

The *Arithmetica* series of books presented the first written exposition of solving algebraic equations. Like Pythagoras, what little we know of Diophantus is through mention of his life and his work in the writings of others. Discoveries of his life and his work continue to this day. As recent as 1968 four books attributed to Diophantus where found through their Arab translations by Qusta ibn Luqa.

Fermat’s Last Theorem was written down for the first time in 1637 by the French Mathematician Pierre de Fermat in the margin of his copy of the one of book *Arithmetica* by Diophantus.

The inscription was in Latin and read: “To divide a cube into two cubes, a fourth power, or in general any power whatever into two powers of the same denomination above the second is impossible. I found an admirable proof of this, but the margin is too narrow to contain it.”

Fermat's Last Theorem is easily stated, but terribly difficult to prove:

Fermat's Last Theorem: There are no positive integers a, b, and c which can satisfy the equation

$$a^n + b^n = c^n$$

where n is greater than 2.

It is known as last Fermat's Last Theorem simply because all of Fermat's other theorems were solved and this was the last to be proved. If fact it was more of a Conjecture that a Theorem, but by convention it has become known as a Theorem.

A complete proof of Fermat's Last Theorem was finally presented in 1993 by British Mathematician Dr. Andrew Wiles, a native of Cambridge England and a professor at Princeton University in New Jersey.

Wiles presented his proof at a mathematics seminar to the acclaim of mathematicians throughout the world, some 358 years after Fermat's wrote his theorem in the margin of his little mathematics book. To complete his proof, Andrew Wiles had worked for nearly a decade in the attic of his home, in near seclusion.

When I was younger I spent several years looking at the previous work and different partial proofs to Fermat's Last Theorem, but wisely begged off the challenge. I have also looked at the Goldbach Conjecture, a conjecture that still captures my interest.

As one historian of mathematics Howard Ewews noted,

> “Fermat’s Last Theorem has the distinction of being the mathematical problem for which the greatest number of incorrect proofs have been published.”

In his vintage BBC Ascent of Man Series, the Polish born British philosopher and scientist, the late Jacob Bronowski described Pythagoras theorem in this way,

“*To this day, the theorem of Pythagoras remains the most important single theorem in the whole of mathematics*.”

The Life and work of both Diophantus and Fermat, as well as the Goldbach Conjecture may be the subject of separate future articles in **Genius Magazine** series.

The Beal Conjecture – How to Win a Million Dollars

The Beal Conjecture is a simple conjecture from number theory that draws its beginning in Fermat's Last Theorem. First proposed in 1993 by the wealthy, American business man and mathematician Andrew Beal, the conjecture is presented as simply as that of Fermat's Last Theorem:

If

$$A^x + B^y = C^z$$

where A, B, C and x, y, z are positive integers greater than 2, then A, B, and C have a common prime factor. The Beal Conjecture can also be stated as

All Fermat – Catalan conjecture solutions will have 2 as an exponent.

Beal beginning in 1997 offered a cash prize of $ 5,000 to a proof or counter example published in a refereed mathematics journal. As of 2013 the Beal Conjecture Prize now stands at $ 1,000,000.

Note the importance of the common prime factor, for it is evident that

$$[a\,(a^m + b^m)]^m + [\,b\,(a^m + b^m)]^m = (a^m + b^m)^{m+1}$$

Good luck in your efforts to prove or disprove the Beal Conjecture!

The Diophantine Algorithm

In Proposition 19 of Book III in his *Arithmetica* series of books Diophantus proved the following Algorithm, which we shall call the Diophantine Algorithm:

$$(a^2+b^2)(c^2+d^2) = (ac + bd)^2 + (ad - bc)^2$$
$$= (ac - bd)^2 + (ad + bc)^2$$

The Diophantine Algorithm shows that the product of two numbers, each of which is a sum of squares, is again the sum of two squares, which can be formed in two ways. You can prove this identity just by multiplying out the right hand side, which is rather tedious and straightforward. Another approach is to use a Gaussian Integer approach.

In number theory a Gaussian Integer is a complex number whose real and imaginary are both integers. The Gaussian Integers, with ordinary addition and multiplication of the complex number i ($i = \sqrt{-1}$) forms a complex integer space that is designated Z[i].

The manner in which we will use Gaussian Integers in our little proof of the Diophantine Algorithm is also related to the number theory concept of quadratic integers.

$$(a^2+b^2) = (a + ib)(a - ib)$$

$$(c^2+d^2) = (c + id)(c - id)$$

This means the identity can be rewritten as

$$(a^2+b^2)(c^2+d^2) = (a + ib)(a - ib)(c + id)(c - id)$$

This means then that

$$(a + ib)(a - ib)(c + id)(c - id) = (a + ib)(c + id)(a - ib)(c - id)$$

from which we find

$$\begin{aligned}(a^2+b^2)(c^2+d^2) &= ([ac - bd] + i[bc + ad])([ac - bd] - i[bc + ad]) \\ &= (ac - bd)^2 + (bc + ad)^2\end{aligned}$$

Similarly, if we shuffle the order,

$$(a^2+b^2)(c^2+d^2) = (a + ib)(c - id)(a - ib)(c + id)$$

from which we find

$$\begin{aligned}(a^2+b^2)(c^2+d^2) &= ([ac + bd] + i[bc - ad])([ac + bd] - i[bc - ad]) \\ &= (ac + bd)^2 + (bc - ad)^2\end{aligned}$$

which is what we set out to show.

The Diophantine Algorithm and Pythagorean Triples

The Diophantine Algorithm can be used to generate Pythagorean Triples that may not necessarily be generated using the Euclid method.

However, not all numbers generated using the Diophantine Algorithm are Pythagorean Triples. Remember, in a Pythagorean Triple, all three numbers must be integers. The Diophantine Algorithm can generate numbers which are not themselves perfect squares of integers.

Let us look at the second case … first. Let

$$p = (a^2+b^2)$$

and

$$q = (c^2+d^2)$$

then

$$pq = (ac - bd)^2 + (bc + ad)^2$$
$$= (ac + bd)^2 + (bc - ad)^2$$

For example, consider the pair of numbers

$$5 = 1^2 + 2^2$$

and

$$10 = 1^2 + 3^2$$

from which we have using the Diophantine Algorithm we find

$$\begin{aligned} pq = 50 &= 5^2 + 5^2 \\ &= 7^2 + 1^2 \end{aligned}$$

We notice immediately that 50 is not a perfect square.

In the first case, we can use the Diophantine Algorithm to construct new Pythagorean Triples using numbers that are produced by the multiplication of perfect squares of integers.

If

$$e^2 = (a^2+b^2)$$

and

$$f^2 = (c^2+d^2)$$

are two Pythagorean Triples (a, b ; e) and (c, d ; f), since

$$e^2 f^2 = (ef)^2 = g^2$$

then

$$g^2 = (ac - bd)^2 + (bc + ad)^2$$
$$= (ac + bd)^2 + (bc - ad)^2$$

are also a pair of Pythagorean Triple with the same hypotenuse length g.

Take as an example the two Pythagorean Triples (3, 4; 5) and (5, 12; 13) from which we get

$$(5^2)(13^2) = (65)^2$$
$$= 33^2 + 56^2$$
$$= 63^2 + 16^2$$

We have generated two Pythagorean Triples (33, 56; 65) and (16, 63; 65).

You can do this forever … Isn't mathematics fun!

Epilogue

In this little article about Pythagoras we have just scratched the surface of his life, the Theorem that bears his name, and the Magic of his Triples.

You may note that some historians write about his numberist or numerological interests and matters incidental to his mathematics. There is mathematics … and then there is other things, like numerology.

I have also not made mention of Pythagoras' interest in the mathematics of music merely because of the focus of this little book. If you are interested in this other stuff – the fluff so to speak of his life – you can read it in other books (these matters are touched upon in passing in Appendix One).

The proof of the Pythagorean Theorem, as well as the Cosine Law is readily understandable to someone who has taken high school mathematics.

How the Fibonacci Sequence is related to Pythagorean Triples is introduced as well as the tie into Fermat's Theorem. The Diophantine Algorithm is also a powerful way to generate new Pythagorean Triples. Perhaps this will wet you're appetite to delve further into the magic of the Pythagorean numbers.

The proof of the Beal Conjecture I leave to my readers. If you are successful don't forget to mention you read this magazine and was inspired in your quest for a proof. Oh, when you receive your prize, don't forget your friends …

If you want to read more about Pythagoras without being overwhelmed by geometry and the finer details of mathematics there is a book by the M.I.T. Historian of Mathematics, the late E. T. Bell titled *The Magic of Numbers.*

I think you would agree that Pythagoras was essentially correct in his belief that "*nature can be described by numbers*". It is left to you, during the course of your journey through life, to discover how well that description is.

Isn't mathematics fun?

Questions

1. Try your hand at producing a dozen Pythagorean Triples.

2. Try your hand for a few weeks at proving Fermat's Last Theorem, then read up about Andrew Wiles' proof. Don't be discouraged. Some of the best mathematical puzzles are simple in their construct. Mathematicians have been driven mad trying to prove this simple theorem.

3. Here is the Question meant to challenge the reader. Consider the set of all Pythagorean triples a, b and c such that b = a+1. Using the algorithm for the Pythagorean Triples provide an expression that solves for q when given a value for p. When p and q are integer numbers, are there many p's and q's such that b = a+1?

4. When we relax the restriction from question 3 that p and q be integer numbers, are there many more p's and q's such that b = a+1?

5. How to find the elements in the Fibonacci Sequence has been shown to you. What do you notice happens to the ratio of two successive elements of the Fibonacci Sequence? What do you notice happens to the inverse of the ratio of two successive elements of the Fibonacci Sequence? Does this number look familiar? What is it called?

6. There is an assumption in the generation of the Fibonacci Sequence that is not born out (pun intended) by real life biology. It has something to do with

flipping a coin. This is an added complication to the problem. Speculate as to how this might change the Sequence.

7. Think about the fact that the Fibonacci Sequence is related to Pythagorean Triples. Try your hand at proving the Raine result. You might begin by looking at the second term, the $(2PIN)^2$ term, and see if it reminds you of something.

8. Instead of using the Gaussian Integer approach to proving the Diophantine Algorithm, derive the identity the old fashion and way.

9. Use the Diophantine Algorithm to generate a half dozen new Pythagorean Triples.

Appendix One: Pythagoras and Pythagoreans

From Encyclopaedia Britannica
Ninth Edition Volume XX, p. 137 – 142.
MDCCCLXXXIII

Pythagoras is one of those figures which have so impressed the imagination of succeeding times that their historical lineaments are difficult to discern through the mystical haze that envelops them. Animated, as it would appear, not merely by the philosophic thirst for knowledge but also by the enthusiasm of an ethico-religious reformer, he became, centuries after his death, the ideal hero or saint, of those who grafted a mystical religious asceticism on the doctrines of Plato. Writings were forged in his name. Lives of him were written which gather up in his person all the traits of the philosophic wise man, and surround him besides with the nimbus of the prophet and wonder-worker.

He is described by his Neo-Platonic biographers as the favourite and even the son of Apollo, from whom he received his doctrines by the mouth of the Delphic priestess. We read that he had a golden thigh, which he displayed to the assembled Greeks in Olympia, and which on another occasion he was seen in Crotona and Metapontus at one and the same time.

He is said to have tamed wild beasts by a word and to have foretold the future with many stories turn upon the knowledge he was reported to retain of his personality and deeds in former states of existence.

Thus, as Zellers truly remarks, the information respecting Pythagoreanism and its founder grows fuller and fuller the farther removed in time it is from its subject. The authentic details of Pythagoras' career, on the other hand, are meagre enough and merely approximate in character.

He was a native of Samos, and the first part of his life may therefore be said to belong to the Ionian seaboard which had already witnessed the first development of philosophic thought in Greece. The exact date of his birth has been variously placed between 586 and 569 B.C. but 582 may be taken as the most probable date. Some of the accounts make him a pupil of Anaximander but such an assertion lies so ready to hand in the circumstances of time and place that we cannot build with any assurance upon the suggested connexion with the Ionic school. It is probable, however, that Pythagoras was aware of their speculations, seeing that he left behind him in Ionia the reputation of a learned and universally informed man.

"*Of all men Pythagoras, the son of Mnesarchus, was the most assiduous inquirer*" says Heraclitus, and then proceeds in his contemptuous fashion to brand his predecessor's wisdom as only eclectically compiled information or polymathy (πολυμαθία).

This accumulated wisdom, as well as most of the tenets of the Pythagorean school, was attributed in antiquity to the extensive travels of Pythagoras, which brought him into contact (so it was said) not only with the Egyptians, the Phoenicians, the Chaldaens, the Jews and the Arabians, but also with the Druids of Gaul, the Persian Magi, and the Brahmans.

But these tails are told of too many of the early philosophers to be received implicitly; they represent rather the tendency of a later age to connect the beginnings of Greek speculation with the hoary religions and priesthood of the East.

There is no intrinsic improbability, however, in the statement that Pythagoras visited Egypt and other countries in the Mediterranean, for travel was then one of the few ways of gathering knowledge. Some of accounts represent Pythagoras as deriving much of his mathematical knowledge from an Egyptian source.

Herodotus traces the doctrine of metempsychosis to Egypt, as well as the practice of burying the dead exclusively in linen garments, but he does not mention any mention of Pythagoras to that country. There is thus little more than conjecture to fill in the first half of the philosopher's life.

The historically important part of his career begins with his emigration to Crotona, one of the Dorian colonies in the south of Italy. Nothing is known with certainty of the reasons that led to this step, which he appears to have taken around the year 529; perhaps the ethical temper that can be traced in the Pythagorean school attracted the founder towards the sterner Dorian character.

At Crotona Pythagoras speedily became the centre of a widespread and influential organization, which seems to have resembled a religious brotherhood or an association for the reformation of society much more than a philosophic school. Pythagoras appears, indeed, in all accounts more as a moral reformer than a speculative thinker or scientific teacher; and it is noteworthy that the only one of the doctrines of the school which is definitely traceable to Pythagoras himself is ethico-mystical doctrine of transmigration.

The aim of the brotherhood was the moral education and purification of the community; and it seems to have been largely based upon a revival of the Dorian ideal of abstinence and hardihood along with the certain other traits of a more definitely religious character, which were probably due to the interest of the mysteries. But many details of life and ritual, such as abstinence from animal food and from beans, celibacy, and even community of goods, have been fathered by the organized asceticism of a later period upon the original followers of Pythagoras.

Ethics, according to the Greek and especially according to the Dorian conception, being inseparably bound up with the general health of the state, we are not surprised to find the Pythagoreans represented as a political league; is it wonderful that their following was among the aristocracy, and that they form the staunchest supporters of the old Dorian constitutions.

It is unfair, however, to speak of the league as primarily a political organization, wide through its political ramifications must latterly have become. Its entanglement with politics was in the end fatal to its existence. The authorities differ hopelessly in chronology, but accordingly to the balance of evidence the first reaction against the Pythagoreans took place in the lifetime of Pythagoras himself after the victory gained by Crotona over Sybaris in the year 510.

Dissensions seem to have arisen about the allotment of the conquered territory and an adverse party was formed in Crotona under the leadership of Cylon. This was probably the cause of Pythagoras' withdrawal to Metapontum, which an almost unanimous tradition assigns as the place of his death in the end of the 6th or the beginning of the 5th century.

The league appears to have continued powerful in Magna Graecia till the middle of the 5th century, when it was violently trampled out by the successive democrats. The meeting-houses of the Pythagoreans were everywhere sacked and burned; mention is made in particular of "the house of Milo" in Crotona, where fifty or sixty of the leading Pythagoreans were surprised and slain. The persecution to which the brotherhood was subjected throughout Magna Graecia was the immediate cause of the spread of the Pythagorean philosophy in Greece proper.

Philolaus, who resided at Thebes in the end of the 5th century, was the author of the first written exposition of the system. Lysis, the instructor of Epaminondas, was another of the refugees. This Theban Pythagoreanism was not without an important influence upon Plato and Philolaus had also disciples in the stricter sense. But as a philosophic school Pythagoreanism became extinct in Greece about the middle of the 4th century. In Italty – where, after a temporary suppression, it attained a new importance in the person of Archytas, ruler of Tarentum – the school finally disappeared about the same time.

Pythagorean Philosophy

The central thought of the Pythagorean philosophy is the idea of the number, the recognition of the numerical and mathematical relations of things. In the naïve speculation of an early age the abstract consideration of these relations was tantamount to asserting their essential existence as the causes of phenomena. Hence the Pythagorean thought crystallized into the formula that all things are number, and that number is the essence of everything. "The Pythagoreans seem", says Aristotle, "to have looked upon the number as the principle and, so to speak, the matter of which existences consist"; and again, "they supposed the elements of the numbers to be the elements of existence, and pronounced the whole heaven to be harmony and number."

“Number”, says Philolaus, “is great and perfect and omnipotent, and the principle and guide of divine and human life.” Fantastical as such a proposition sounds, we may still recognize the underlying truth that prompted it if we reflect that it is number or definite mathematical relation that separates one thing from another and so in a sense makes them things. Without number and the limitation which number brings there would be only chaos and the illimitable, a thought abhorrent to the Greek mind. Number, then, is the principle of order, the principle by which a cosmos or ordered world subsists. So we may perhaps render the thought that is crudely and sensuously expressed in the utterance of the school. They found the chief illustrations, or rather grounds, of their position in the regular movements of the heavenly bodies and in the harmony of musical sounds, the dependence of which on regular mathematical intervals they were apparently the first to discover. The famous theory of the harmony of the spheres combines both ideas: the seven planets are the seven golden chords of the heavenly heptachord.

Immediately connected with their central doctrine is the theory of opposites held by the Pythagoreans. Numbers are divided into odd and even, and from the combination of odd and even the numbers themselves (and therefore all things) seem to result. The odd number was identified with the limited, the even with the unlimited, because even numbers may be perpetually halved, whereas the odd numbers (at least the earlier ones), being without factors, seem to stand in solid singleness. All things, accordingly, were derived by the Pythagoreans from the combination of the limited and the unlimited; and it is the harmony with the Greek spirit that the place of honour is accorded to the odd or limited.

Following the same thought, they developed a list of ten fundamental oppositions, which roughly resembles the tables of categories framed by the later philosophers:

(1) Limited and unlimited
(2) odd and even
(3) one and many
(4) right and left
(5) masculine and feminine
(6) rest and motion
(7) straight and crooked
(8) light and darkness
(9) good and evil
(10) square and oblong

The arbitrariness of the list and the mingling of mathematical, physical and ethical contrasts are characteristic of the infancy of the speculation. The union of opposites in which consists the existence of things in harmony; hence the expression already quoted that the whole heaven or whole universe is harmony.

But it is to be noted that interpretations of Pythagoreanism which represent the whole system as founded on the unity and duality, and suppose this to have been explicitly identified with the opposition of form and matter, of divine activity and passive material, must be unhesitatingly rejected as betraying on the surface their post-Platonic origin. Still more is this the case when in Neo-

Platonic fashion they go on to derive this original opposition from the supreme Unity of God. The further speculations of the Pythagoreans on the subject of number rest mainly on analogies, which often become capricious and tend to lose themselves at last in a barren symbolism. The decade, on the basis of the numerical system, appeared to them to comprehend all other numbers in itself, and to it are applied, therefore, the epithets quoted above the energy are numbers in general. Similar language is held of the number "four", because it is the first square number and is the potential decade (1+2+3+4=10); Pythagoras is celebrated for the discovery of the holy τετρακτύς, "the fountain and root of ever-living nature." "Seven" is called παρθένος and 'Αθήνη , because within the decade it has neither factors nor products. "Five", on the other hand, signifies marriage, because it is the union between the first masculine and feminine number (3 + 2, unity being considered as a number apart). The thought already becomes more fanciful when "one" is identified with reason, because it is unchangeable; "two" with opinion, because it is unlimited and indeterminate; "four" with justice, because it is the first square number, the product of equals.

More legitimate is their application of number to geometry, according to which "one" was identified with the point, "two" with the line, "three" with the surface, and "four" with the cube". In the history of music, the Pythagorean school is also of considerable importance from the development which the theory of octave owes to its members; according to some accounts the discovery of harmonic system is due to Pythagoras himself.

As already mentioned, the movements of the heavenly bodies formed for the Pythagoreans an illustration on a grand scale of the truth of their theory. Their cosmological system is also interesting on account of peculiarities which count it out from the current conceptions of antiquity and bring it curiously near to the modern theory. Conceiving the universe, like many early thinkers, as a sphere, they placed in the heart of it the central fire , to which they gave the name Hestia, the hearth or altar of the universe, the citadel or throne of Zeus. Around this move the ten heavenly bodies – farthest off the heaven of the fixed stars, then the five planets known to antiquity, then the sun, the moon, the earth and lastly the counter-earth (άντιχθωυ), which revolves between the earth and the central fire and thus completes the scared decade. Revolving along with the earth , the last-mentioned body is always interposed as a shield between us and the direct rays of the central fire. Our light and heat come to us indirectly by way of reflexion from the sun. When the earth is on the same side of the central fire as the sun, we have day; when it is on the other side, night. This attribution of the changes of day or night to the earth's own motion led up directly to the true theory, as soon as the central fire and the counter-earth was dispensed with. The counter-earth became the western hemisphere, and the earth revolved around its own axis instead of round an imaginary centre. But, as appears from above, the Pythagorean astronomy is also remarkable as having attributed a planetary motion to the earth instead of making our globe the centre of the universe. Long afterward, when the church condemned the theory of Copernicus, the indictment that lay against it was its heathen and "Pythagorean" character.

The doctrine which with the memory of mankind associates most closely with Pythagoras' name is that of the transmigration of souls – METEMPSYCHOSIS (q.v.). Though evidently of great importance for Pythagoras himself, it does not stand in any very obvious connexion with his philosophy proper. He seems to have adopted the idea from Orphic Mysteries.

The bodily life of the soul, according to the doctrine, is an imprisonment suffered for sins committed in a former state of existence. At death the soul reaps what it has sown in the present life. The reward of the best is to enter the cosmos, or the higher or purer regions of the universe, while the direst crimes receive their punishment in Tartarus. But the general lot is to live afresh in a series of human or animal forms, the nature of the bodily prison being determined in each case by the deeds done in the life just ended. This is the same doctrine of retribution and purificatory wandering which meets us in Plato's mythical description of a future life. They are borrowed by him in their substance from the Pythagoreans or from a common source in the Mysteries. In accordance with this religious view of life as a stage of probation were the ethical precepts of the school, inculcating reverence towards the gods and to parents, justice, gentleness, temperance, purity of life, prayer, regular self-examination, and the observance of various ritual requirements.

Connecting its ethics in this way with religion and the idea of a future life, the Pythagorean societies had in them from the beginning a germ of asceticism and contemplative mysticism, which it was left for a later age fully to develop.

The Pythagorean life was destined to survive the peculiar doctrines of the Pythagorean philosophy and to be grafted on later philosophic ideas. The asceticism which characterized it appears in the 4th century BC in close connection with the Orphic Mysteries; and the "Pythagoreans" of that time are frequently the butts of the New Athenian Comedy.

In the Alexandrian period the Pythagorean tradition struck deeper roots; in Alexandria and elsewhere schools of men arose calling themselves Pythagoreans, but more accurately distinguished by modern criticism as Neo-

Pythagoreans, seeing that their philosophical doctrines are evidently derived in varying proportions from Plato, Aristotle, and the Stoics.

In general it may be said that they develop the mystic side of the Platonic doctrine; and only so far as this is connection with the similar speculations of Pythagoras can they claim to be followers of the latter. Hence men like Plutarch, who personally prefer to call themselves Platonists, may also be considered within the scope of this Pythagorean revival.

The link that really connects these Neo-Pythagoreans with the Samian philosopher and distinguishes them from the other schools of their time is their ascetic ideal of life and their preoccupation with religion. In religious speculation they paved the wave for the Neo-platonic conception of God as immeasurably transcending the world; and their thirst for prophecies, oracles, and signs they gave expression to the prevalent longing for a supernatural revelation of the divine nature and will.

The asceticism of the Jewish sect of the Essenes seems, as Zellers contends, to be due to a strong infusion of Neo-Pythagorean elements. At a still later period Neo-Pythagoreanism set up Pythagoras and Apollonius of Tyana not only as ideals in a philosophic life but as prophets and wonder-workers in immediate communication with another world, and the details of their "lives" it is easy to read the desire to emulate the narratives of the Gospels. The Life of Apollonius by Philostratus, which is for the most part an historical romance, belongs to the 3rd Christian century.

Pythagorean Geometry

As the introduction of geometry into Greece is by common consent attributed to Thales, so all are agreed that the Pythagoras is due the honour of having raised mathematics to the rank of a science. We know that the early Pythagoreans published nothing and that, moreover, they referred all their discoveries back to their Master. Hence it is not possible to separate his work from that of his early disciples, and we must therefore treat the geometry of the early Pythagorean school as a whole. We know that Pythagoras made numbers the basis of his philosophical system, as well physical and metaphysical, and that he united the study of geometry with that of arithmetic.

The following statements have been handed down to us:

(a) Aristotle (Met., i., 5, 985) says "the Pythagoreans first applied themselves to mathematics, a science which they improved; and, penetrated with it, they fancied that the principles of mathematics were the principles of all things."

(b) Eudemus informs us that "Pythagoras changed geometry into the form of a liberal science, regarding its principles in a purely abstract manner, and investigated its theorems from the immaterial and intellectual point of view." [1]

(c) Diogenes Laertius (viii. 11) relates that "it was Pythagoras who carried geometry to perfection, after Moeris [2] had first found out the principles of the elements of that science, as Anticlides tells us in the second book of the *History of Alexander*; and the part of the science to which Pythagoras applied himself above all others was arithmetic."

(d)According to Aristoxenus, the musician, Pythagoras seems to have esteemed arithmetic above everything and to have advanced it by diverting it from the service of commerce and by likening all things to numbers." [3]

(e) Diogenes Laertius (viii. 11) reports on the same authority that Pythagoras was the first person who introduced measures and weights amongst the Greeks.

(f) He discovered the numerical relations of the musical scales (Diogenes Laertius, viii. 11)

(g) Proclus [4] says that "the word 'mathematics' originated with the Pythagoreans."

(h) We learn also from the same authority [5] that Pythagoras made a fourfold division of mathematical science, attributing one of its parts to the "how many" (τό πόσου) and the other to the "how much" (τό πηλίκου); and they assigned to each of these parts a twofold division. They said that discrete quantity or the "how many" is either absolute or relative, and that continued quantity or the "how much" is either stable or in motion. Hence they laid down that arithmetic contemplates the discrete quantity which subsists by itself, but music that which is related to another; and that geometry considers continued quantity so far as it is immovable, but that astronomy (ή σφαιρική) contemplates continued quantity so far as it is of a self-motive nature.

(i) Diogenes Laertius (viii. 25) states, on the authority of Favorinus, that Pythagoras " employed definitions in the mathematical subjects to which he applied himself."

The following notices of the geometrical work of Pythagoras and the early Pythagoreans are also preserved;

(1) The Pythagoreans define a point as "unity having position" (Procl., *op. cit.*, p.95).

(2) They considered a point as analogous to a monad, a line to a duad, a superficies to the triad, and a body to a tetrad (*ib.*, p. 97)

(3) They showed that a plane is completely filled by six equilateral triangles, four squares, or three regular hexagons (*ib.*, p. 305)

(4) Eudemus ascribes to them the discovery of the theorem that the interior angles of a triangle are equal to two right triangles, and gives their proof, which was the same as that in Euclid I.32 [5] (ib. 379).

(5) Proclus informs us in his commentary on Euclid I.44 that Eudemus says that the problems concerning the application of areas – where the term "application" is not to be taken in the restricted sense (παραβολή), in which it is use in this proposition, but also in the wider signification, embracing ύπερβολή and έλλειψις in which it is used in Book VI. Props 28, 29 – are old, and inventions of the Pythagoreans [6] (ib. 419).

(6) This is confirmed by Plutarch, [7] who says, after Apollodorus, that Pythagoras sacrificed an ox on finding the geometrical diagram, either the one relating tot the hypotenuse, namely, that the square on it is equal to the sum of the squares of the sides, or that relating to the problem concerning the application of an area. [8]

(7) Plutarch [9] also ascribes to Pythagoras the solution of the problem, to construct a figure equal to one and similar to another figure.

(8) Eudemus states that Pythagoras discovered the construction of the regular solids (Procl., *op. cit.* p.65)

(9) Hippasus, the Pythagorean, is said to have perished in the sea on account of his impiety, inasmuch as he boasted that he first divulged the knowledge of the sphere with the twelve pentagons (the inscribed ordinate dodecahedron); Hippasus assumed the glory of the discovery to himself, whereas everything belonged to Him –

"for thus they designate Pythagoras, and not not call him by name."[10]

(10) The triple interwoven triangle, or pentagram – star-shaped regular pentagon – was used as a symbol of recognition by the Pythagoreans and was called by them "health" (ὑγιεία).[11]

(11) The discovery of the three squares (Euclid I.47), commonly called the "theorem of Pythagoras," is attributed to him by many authorities, of whom the oldest is Vitruvius.[12]

(12) One of the methods of finding right triangles whose sides can be expressed in numbers (Pythagorean triangles) – that setting out from the odd numbers – is referred to Pythagoras by Heron of Alexandria and Proclus.[13]

(13) The discovery of irrational quantities is ascribed to Pythagoras by Eudemus (Procl., *op. cit.* p.65).

(14) The three propositions – arithmetical, geometrical and harmonical – were known to Pythagoras.[14]

(15) Iamblichus[15] says, "Foremrly in the time of Pythagoras and the mathematicians under him, there were three means only – the arithmetical, the geometrical and the third in order which was known by the name sub-contrary (ὑπεναντία), but which Archytas and Hippasus designated the harmonical, since it appeared to include the ratios concerning harmony and melody."

(16) The so called most perfect or musical proportion, eg., 6:8::9:12, which comprehends in it all the former ratios, according to Iamblichus,[16] is said to be an invention of the Babylonians and to have been first brought into Greece by Pythagoras.

(17) Arthmetrical progressions were treated by the Pythagoreans, and it appeareds from a passage in Lucian that Pythagoras himself had considered the special case of the *triangular* number: Pythagoras asks some one, “How do you count?” he replies “One, two, three, four” Pythagoras, interrupting, says, “Do you see? What you take to be four, that is ten and a perfect triangle and our oath.” [17]

(18) The odd numbers were called by the Pythagoreans “gnomons”, [18] and were regarded as generating, in as much as by the addition of successive gnomons – consisting each of an odd number of unit squares – to the original square unit or monad the square form was preserved.

(19) In like manner, if the simplest oblong (έτερόμηκες), consisting of two units squares or monads in juxtaposition, be taken and four unit squares be placed after it after the manner of a gnomon, and then in like manner six, eight … unit squares be placed in succession, the oblong form will be preserved.

(20) Another of his doctrine was, that of all solid figures the sphere was the most beautiful, and of all plane figures the circle. [19]

(21) According to Iamblichus the Pythagoreans are said to have found the quadrature of the circle. [20]

On examining the purely geometrical work of Pythagoras and his early disciples, as given in the preceding extracts, we observe that it is much concerned with the geometry of areas, and we are indeed struck by its Egyptian character. This appears in the theorem (3) concerning the filling of a plane with regular figures – for walls and floors covered with tiles of various colours were common in Egypt; in the construction of the regular solids (8), for some of them are found in Egyptian architecture; in the problems concerning their applications of areas (5); and lastly in the theorem of Pythagoras (11), coupled with his rule for the construction of right triangles (12).

We learn from Plutarch that the Egyptians were acquainted with the geometrical fact that a triangle whose sides contain three, four and five parts is right-angled, and that the square of the greatest side is equal to the squares of the sides containing the right angle. It is probable too that this theorem was known to them in the simple case where the right-angle triangle is isosceles, inasmuch as it would be at once suggested by the contemplation of a floor covered with square tiles – the square of the diagonal and the sum of the squares on the side contain each four of the right-angled triangle s into which one of the squares is divided by its diagonal. It is easy not to see how the problem to construct a square, which shall be equal to the sum of two squares could, in some cases, be solved numerically. From the observation of a chequered board it would be perceived that the element in the successive formation of squares is the gnomon or carpenter’s square.

Each gnomon consists of an odd number of squares, and the successive gnomons correspond to the successive odd numbers, and include, therefore, all odd squares. Suppose, now, two squares are given, one consisting of sixteen and the other of nine unit squares, and that it is proposed to form from them another square. It is evident that the square consisting of nine unit squares can take the form of the fourth gnomon, which being placed around the former square, will generate a new square containing twenty-five unit squares. Similarly it may have been observed that the twelfth gnomon, consisting of twenty-five unit squares, could be transformed into a square each of whose sides contains five units, and thus it may have been conversely that the latter square, by taking the gnomic or generating form with respect to the square on twelve units as base, and would produce the square of thirteen units, and so on.

This method required only to be generalized in order to enable Pythagoras to arrive at this rule for finding right-angle triangles whose sides can be expressed in numbers, which, we are told, sets out from the odd numbers. The nth square together with the nth gnomon forms the (n+1)th square; if the nth gnomon contains m^2 unit squares, m being an odd number, we have $2n+1 = m^2$, therefore $n = \frac{1}{2}(m^2 - 1)$, which gives the rule of Pythagoras.

The general proof of Euclid I.47 is attributed to Pythagoras, but we have the express statement of Proclus (Procl., *op. cit.* p. 426) that this theorem was not proved in the first instance as it is in the *Elements*.

The following simple and natural way of arriving at this theorem is suggested by Bretschneider after Camerer. [21] A square can be dissected into the sum of two squares and two equal rectangles, as in Euclid II.4; these two rectangles can, by drawing their diagonals, be decomposed into four equal right-angled triangles, the sum of the sides of each being equal to the side of the square; again, these four right triangles can be placed so that a vertex of each shall be one of the corners of the square in such a way that a greater and less side are in continuation. The original square is thus dissected into the four triangles as before and the figure within, which is the square on the hypotenuse. This square, therefore, must be equal to the sum of the squares on the sides of the right-angle triangle.

It is well known that the Pythagoreans were more occupied with the construction of regular polygons and solids, which in their cosmology played an essential part as the fundamental forms of the elements of the universe. We can trace the origin of these mathematical speculations in the theorem (3) that "the plane around a point is completely filled by six equilateral triangles, four squares or three regular hexagons."

Plato also makes the Pythagorean Timaeus explain – "Each straight-lined figure consists of triangles, but all triangles can be dissected into rectangular ones that are either isosceles or scalene. Among the latter the most beautiful is that out of the doubling of which an equilateral arises, or in which a square of the greater perpendicular is three times that of the smaller, or which the smaller perpendicular is half the hypotenuse. But two or four right-angled isosceles triangles, properly put together, form the square; two or six of the most beautiful scalene right-angled triangles form the equilateral triangle; and out of these two figures arise the solids which correspond with the four elements of the real world, the tetrahedron, octahedron, icosahedron, and the cube" [22] (*Timaeus* 53, 54, 55).

The construction of the regular solids is distinctly ascribed to Pythagoras himself by Eudemus (8). Of these five solids three – the tetrahedron, the cube and the octahedron – were known to the Egyptians and are to be found in their architecture.

Let us examine what is required for the construction of the two other solids – the icosahedron and the dodecahedron. In the formation of the tetrahedron, three, and in that of the octahedron four, equal equilateral triangles have been placed with a common vertex and adjacent sides coincident; and it is known that if six such triangles were placed round a common vertex with their adjacent sides coincident, they would lie in a plane, and that, therefore, no solid can be formed in that manner from them. It remained, then, to try whether five such equilateral triangles could be placed at a common vertex in like manner; on trial it would be found that they could be so placed, and that their base would form a regular pentagon. The existence of a regular pentagon would thus become known.

It was also known from the formation of the cube that three squares could be placed in a similar manner with a common vertex; and that, further, if three equal and regular hexagons were placed round a common vertex with adjacent sides coincident, they would form a plane. It remained in this case too only to try whether three regular pentagons could be placed with a common vertex and in a similar way; this on trial would be found possible and would lead to the construction of the regular dodecahedron, which was the regular solid last arrived at.

We see that the construction of the regular pentagon is required for the formation of each of these two regular solids, and that, therefore, it must have been a discovery of Pythagoras. If we examine now what knowledge of geometry was required for the solution of this problem, we shall see that it depends on Euclid IV.10, which is reduced to Euclid II.11, which problem is reduced to the following: To produce a given straight line so that the rectangle under the whole line thus produced and the produced part shall be equal to the square on the given line, or, in the language of the ancients; to apply to a given straight line a rectangle which shall be equal to the given area – in this case the square on the given line – and which shall be *excessive* by a square. Now it is to be observed that the problem is solved in this manner by Euclid (VI, 30, 1st method), and that we know on the authority of Eudemus that the problems concerning the application of areas and their *excess* and *defect* are old, and inventions by the Pythagoreans (5).

Hence the statements of Iamblichus concerning Hippasus (9) – that he divulged the sphere with the twelve pentagons – and of Lucian and the scholiast on Aristophanes (10) – that the pentagram was used as a symbol of recognition amongst the Pythagoreans – become of greater importance.

Further, the discovery of irrational magnitudes is ascribed to Pythagoras by Eudemus (13), and this discovery has been ever regarded as one of the greatest of antiquity. It is commonly assumed that Pythagoras was lead to this theory from the consideration of the isosceles right angle triangle.

It seems to the present writer, however, more probable that the discovery of the incommensurable magnitudes was rather owing to the problem: To cut a line extreme and mean ratio. From the solution of this problem it follows at once that, if on the greater segment of a line so cut a part to be taken equal to the less, the greater segment, regarding a new line, will be cut in a similar manner; and this process can be continued without end.

On the other hand, if a similar method is adopted in the case of any two lines which can be represented numerically, the process would end. Hence would arise the distinction between commensurable and incommensurable quantities. A reference to Euclid X.2 will show that the method above is the one used to prove that two magnitudes are incommensurable; and in Euclid X.3 it will be seen that the greatest common measure of two commensurable magnitudes is found by this process of continued subtraction.

It seems probable that Pythagoras, to whom is attributed one of the rules for representing the sides of right-angled triangles in numbers, tried to find the sides of an isosceles right-angled triangle numerically, and that, failing in the attempt, he suspected that the hypotenuse and a side had no measure. He may have demonstrated the incommensurability of the side of the square and its diagonal.

The nature of the old proof – which consisted of a *reductio ad absurdun*, showing that, if the diagonal be commensurable with the side, it would follow that the same number would be odd and even [23] – makes it more probable, however, that this was accomplished by his successors. The existence of the irrational as well as the regular dodecahedron appears to have been regarded by the school as one of their chief discoveries, and to have been preserved as a secret; it is remarkable, too, that a story similar to that told by Iamblichus of Hippasus is narrated of the person who first published the idea of the irrational, namely, that he suffered shipwreck, &c. [24]

Eudemus ascribes the problems concerning the application of the figures to that of Pythagoras. The simplest case of the problems, Euclid VI. 28, 29 – those, namely, in which the given parallelogram is a square – corresponds to the problem: to cut a given straight line internally or externally so that the rectangle under the segments shall be equal to a given rectilineal figure. The solution of this problem – in which the solution of a quadratic equation is implicitly contained – depends on the problem, Euclid II.14, and the theorems, Euclid II. 5 and 6, together with the theorem of Pythagoras. It is probable that the finding of a men proportional between two given lines, or the construction of a square which shall be equal to a given rectangle, is due to Pythagoras himself.

The solution of the more general problem, Euclid VI. 25, is also attributed to him by Plutarch (7). The solution of this problem depends on that of the particular case and on the application of the areas; it requires, moreover, a knowledge of the theorems: Similar rectilineal figures are to each other as the squares of their homologous sides (Euclid VI. 20); and, if three lines are in geometrical proportion, the first is to the third as the square on the first is to the square of the second. Now Hippocrates of Chios, about 440 B.C., who was instructed in geometry by the Pythagoreans, possessed this knowledge. We are justified, therefore, in ascribing the solution of the general problem, if not (with Plutarch) to Pythagoras, at least to his early successors.

The theorem of similar polygons are to each other in the duplicate ratio of their homologous sides involves a first sketch, at least, of the doctrine of proportion and the similarity of the figures. [25] That we owe the foundation and development of the doctrine of the proportion to Pythagoras and his school is confirmed by the testimony of Nicomachus (14) and Iamblichus (15 and 16). From these passages it appears that the early Pythagoreans were acquainted, not only with the arithmetical and geometrical means between two magnitudes, but also with their harmonical means, which was called "subcontrary".

The Pythagoreans were much occupied with the representation of numbers by geometrical figures. These speculations originated with Pythagoras, who was acquainted with the summation of the natural numbers, the odd numbers, and the even numbers, all of which are capable of geometrical representation. See the passage in Lucian (17) and the rule for finding Pythagorean triangles (12) and the observations thereon *supra*.

On the other hand, there is no evidence to support the statement of Montcula that Pythagoras laid the foundation of the doctrine of *isoperimetry*, by proving that of all figures having the same perimeter the circle is the greatest, and that of all solids having the same surface the sphere is the greatest. We must also deny to Pythagoras and his school a knowledge of the conic sections, and in particular of the quadrature of the parabola, attributed to him by some authors; and we have noticed the misconception which gave rise to this erroneous inference.

Let us now see what conclusions can be drawn from the foregoing examination of the mathematical work of Pythagoras and his school, and thus from an estimate of the state of geometry about 480 B.C.

First, as to *matter*: It forms the bulk of the first two books of Euclid, and includes a sketch of the doctrine of proportion – which was probably limited to commensurable magnitudes – together with some of the contents of the sixth book. It contains too the discovery of the irrational (άλογον) and the construction of the regular solids, the later requiring the description of certain regular polygons – the foundation, in fact, of the fourth book of Euclid.

Secondly, as to *form*; The Pythagoreans first severed geometry from the needs of practical life, and treated it as a liberal science, giving definitions and introducing the manner of the proof which has ever since been in use. Further, they distinguished between *discrete* and *continuous* quantities, and regarded geometry as a branch of mathematics, of which they made the fourfold division that lasted to the Middle Ages – the *quadrivium* (fourfold way to knowledge) of Boetius and the scholastic philosophy. And it may be observed that the name of "mathematics," as well as "philosophy," is ascribed to them.

Thirdly, as to *method*; One chief characteristic of the mathematical work of Pythagoras was the combination of arithmetical with geometry. The notions of an equation and a proportion – which are common to both, and contain the first germ of algebra – were introduced among the Greeks by Thales.

These notions, especially the latter, were elaborated by Pythagoras and his school, so that they reached the rank of a true scientific method in their theory of proportion. [26]

To Pythagoras, then, is due the honour of having supplied a method which is common to all branches of mathematics,and in this respect he is fully comparable to Descartes, to whom we owe the decisive combination of algebra with geometry.

See C.A. Bretschneider, *Die Geometrie u. die Geometer vor Euklides* (Leipzig, 1870); H. Hankel, *Zur Geschichte der Mathematik* (Leipzig, 1874); F. Hoefer, *Histoire des Mathématiques* (Paris, 1874); G. J. Allman, "Greek Geometry from Thales to Euclid", in Hermathena, Nos. V., vii., and x. (Dublin 1877, 1881 and 1884); M. Cantor, *Vorlesungen über Geschicte der Mathematik* (Leipzig, 1884). The recently published *Short History of Greek Mathematics* by James Gow (Cambridge, 1884) will be found a convenient compilation.

Notes:

1. Proclus Diadochus, *In primum Euclidis Elementorum librum Commentarii*, ed. Friedlein, p. 65

2. Moeris was a king of Egypt who, Herodotus tells us, lived 900 years before his visit to that country.

3. Aristox. Fragm., ap. Stob., Eclog. Phys., 1. 2,6

4. *Op. cit.*, p. 35

5. We learn, however, from a fragment of Geminus, which has been handed down by Eutocius in his comments on the Conics of Apollonius (Apoll. Conica, ed. Halleius, p.9), that the ancient geometers observed two right angles in each species of triangle, in the equilateral first, then in the isosceles, and lastly in the scalene, whereas later writers proved the theorem generally thus – "The three internal angles of every triangle are equal to two right angles."

6. The words of Proclus are interesting. "According to Eudemus the inventions respecting the *application*, *excess* and *defect* of areas are ancient, and are due to the Pythagoreans. Moderns, borrowing these names, transferred them to the so-called conic lines, the parabola, the hyperbola, the ellipse, as the older school, in their nomeclature concerning the description of areas *in plano* on a finite right line, regarding the terms thus: – An area is said to be applied (παραβάλλειν) to a given right line when an area equal in content to some given one is described thereon; but when the base of area is greater than the given line, then the area is said to be in *excess* (ύπερβάλλειν); but when the base is less, so that some part of the given line lies without the described area, then the area is said to be in *defect* (έλλείπειν). Euclid useds

in this way the sixth book the terms *excess* and *defect*. …. The term application (παραβάλλειν), which we owe to the Pythagoreans, has this signification."

7. *Non posse suaviter viv sec*. Epicurum, c. xi.

8. Είτε πρόβλημα περί τού χωρίου τής παραβολής. Some authors, rendering the last five words "concerning the area of the parabola," have ascribed to Pythagoras the quadrature of the parabola, which was one of the great discoveries of Archimedes.

9. *Symp*. viii., Quaest. 2, c.4.

10. Iamblichus, De Vit. Pyth., c. 18, s.88.

11. Lucian, *Pro Lapsu in Salut*., s.5; also school. On Aristoph., *Nub*., 611. That the Pythagoreans used such symbols we learn from Iamblichus (*De. Vit. Pyth*., c. 33, ss. 237 and 238). This figure is referred to Pythagoras himself, and in the Middle Ages was called Pythagorae figura; even so late as Paracelsus it was regarded by him as a symbol of health. It is said to have obtained its special name from the letters υ,γ,ι,θ (= εί), α having been written at its prominent vertices.

12. *De Arch.* , ix., Praef., 5, 6, 7. Amongst other authorities are Diogenes Laertius (viii. 11), Proclus (*op. cit.*, p.426) and Plutarch (*ut sup.*, 6). Plutarch, however, attributes to the Egyptians the knowledge of this theorem in the particular case where the sides 3,4 and 5 (*De Is. Et Osir.*, c.56).

13. Heron Alex., Geom. Et Stereom. Rel., ed. F. Hultsch, pp. 56, 146; Procl. op.cit., p. 428. The method of Pythagoras is as follows; - he took an odd number as the lesser side; then, having squared this number and diminished the quare by unity, he took half the remainder as the greater side, and by adding unity to this number obtained the hypotenuse, e.g. 3,4,5; 5,12,13.

14. Nico. Ger., *Introd. Ar*., c. xxii.

15. *In Nicomachi Arithmeticam*, ed. S. Tennulius, p. 141.

16. *Op. cit*., p. 168. As an example of this proportion Nicomachus and, after him, Iamblichus give the numbers 6, 8, 9, 12, the harmonical and arithmetical means between the two numbers themselves (a : (2ab)/(a+b) :: (a+b)/2 :2). Iamblichus further relates (l.c.) that many Pythagoreans made use of this proportion, as Aristaeus of Crotona, Timaeus of Locri, Philolaus and Archytas of Tarentum, and many others, , and after them Plato in his *Timaeus* (see Nicom., *Inst. Arithm*., ed. Ast p. 153, and *Animadversiones*, pp. 327 –329; and Iambl., *op. cit*., p. 172 sq.).

17. βίων πράσις, 4, volo. I. P. 317, ed. C. Jacobitz.

18. Γυώμων means that by which anything is known, or "criterion"; its oldest concrete signification seems to be the carpenter's square (norma) by which a right angle is known. Hence it came to denote a perpendicular, of which, indeed was the archaic name (Proclus, *op. cit*., p. 283). Gnomon is also an instrument for measuring altitudes, by means of which the meridian can be found; it denotes, further the index or style of a sun-dial, the shadow of which points out the hours. In geometry it means the square or rectangle about the diagonal of a square or rectangle, together with the two complements, on account of the resemblance of the figure to a carpenter's square; and then, more generally, the similar figures with regards to any parallelogram, as defined by Euclid II. Def.2. Again, in a still more general signification, it means the figure which, being added to any figure, preserves the original form. See Heron, *Definitiones* (59). When gnomons are added successively in this manner to a square monad, the first gnomon may be regarded as that consisting of three square monads, and is indeed the constituent of a Greek

fret; the second of five square monads, &c.; hence we have the gnomic numbers.

19. Diog. Laert., *De Vit. Pyth.*, viii. 19.

20. Simplicus, *In Aristotelis Physicorum libros quattuor priores Commentaria*, ed. H. Diels, p.60.

21. See Bretsch., Die Geom. Vor Euklides, p. 82; Camerer, Euclidis Elem., vol. i. p. 444, and the references given there.

22. The dodecahedron was assigned to the fifth element, *Quinta pars, aether*, or, as some think, to the universe (See Philolaus).

23. For this proof, see Euclid X. 117; see also Aristot., Analyt. Pr., I, c. 23 and c. 44.

24. Knoche, *Untersuchungen über die neu aufgefundenen Scholien des Proklus Diadochus zu Euclid's Elementen*, pp. 20 and 23, Hereford, 1865.

25. It is agreed on all hands that there are two theories were treated at length by Pythagoras and his school. It is almost certain, however, that the theorems arrived at were proved for commensurable magnitudes only, and were assumed to hold good for all. The Pythagoreans themselves seem to have been aware that their proof were not rigorous, and were open to serious objection; in this we may have the explanation of the secrecy which was attached by them to the idea of the incommensurable and to the pentagram which involved, and indeed represented, that idea. Now it is remarkable that the doctrine of proportion is twice treated in the *Elements* of Euclid – first, in a general manner, so as to include incommensurables, In Book V., which tradition ascribes to Eudoxus, and then arithmetically in Book VII., which as Hankel has supposed, contains the treatment of the subject by the older Pythagoreans.

26. Proportion was not regarded by the ancients merely as a branch of arithmetic. We learn from Proclus that “Eratosthenes looked on proportion as the bond of mathematics” (op. cit., p. 43). We are also told in an anonymous scholium on the *Elements* of Euclid which Knoche attributes to Proclus, that the fifth book, which treats proportion, is common to geometry, arithmetic, music, and, in a word, to all mathematical science. And Kepler, who lived near enough to the ancients to reflect the spirit of their methods, says that one part of the geometry is concerned with the comparison of figures and quantities, when proportion arises. He also adds that arithmetic and geometry afford mutual aid to each other, and that they cannot be separated.

Appendix Two: Introduction to the Fibonacci Sequence

Leonardo da Pisa, (1170 to 1240) was also known as Fibonacci – the son of Bonacci.

Fibonacci is considered the first great western mathematician after the decline of Greek science. In 1202 Leonardo da Pisa wrote *Liber Abaci* (Book of the Abacus) in which he introduced modern symbols for numbers (0 to 9). In *Liber Abaci* Leonardo da Pisa provided an arithmetic base for these symbols, and a basic theory of equations for European thinkers. The symbols were borrowed from Indian and Arabic sources and replaced the ponderous roman numeration (e.g. I, II, III, IV, V, VI, etc).

Da Pisa's later book *Practica Geometriae*, published in 1225, is mainly inspired by Greek mathematics; it contains theorems from Euclid's *Elements* and also Heron's formula for the area of a triangle. The reintroduction and clarification of these principles was important in establishing a solid foundation of indisputable formulae on which mathematics could rest.

Leonardo da Pisa's striking ability in solving algebraic equations of higher degree than quadratic further emerged in his works entitled *Liber Quadratorum* and *Flos*, both of which also appeared in 1225.

Leonardo da Pisa (Fibonacci) initiated the tradition of the *Maestri d'Abaco*, which implies an expert in practical geometry, algebra and arithmetic. This traditional flourished in Pisa beginning in the 14th century.

Fibonacci's successors at the University of Pisa included many of the great mathematical minds of the time including Cardano, Ferrari, Pacioli , Tartaglia and Galileo Galilei.

The Fibonacci Sequence originated in a mathematics problem presented by Leonardo da Pisa in his book *Liber Abaci*.

Rabbits breed rapidly. It is assumed a breeding male and female pair of rabbits produce a pair of young rabbits every month and that the new born rabbits become adults in two months and produce, at this time, another pair of rabbit.

Starting with an adult pair, determine the size of the rabbit colony, after the first month, second month, third month, etc.

From this we get the Fibonacci Sequence

Fibonacci Sequence = 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55,…

Appendix Three: Half Man, Half Myth

Chapter Ten: The Magic of Numbers

By Eric Temple Bell

As befits a sage who knew he was great and imagined he was half divine, the legendary Pythagoras is an austere figure, always wise, always temperate, and never once relaxing into anything half so human as the engaging rascalities of Thales. When asked by his disciples what they should call him, Pythagoras did not arrogate to himself the title of wise man, but requested that he be called simply a philosopher – a lover of wisdom. This sincere modesty is as exact an index as anything, in the devious life of this philosopher, to his almost fanatical devotion to knowledge in the presence of the knowable.

It may be stated once for that this extraordinary spirit is known to us only through legends and traditions, for none of which is there any contemporary documentary evidence. Even his dates are in dispute, but 569-500 B.C., accepted by many scholars, are frequently given. Slight adjustments of both dates seem necessary to fit the chronology of his life, and these are usually assumed by the chroniclers without comment.

Though it is unlikely that anything fully reliable will ever be discovered about Pythagoras as a man, a great deal is known of what his successors thought of him. As in the case of Thales, these unsubstantiated judgments may be more revealing than any official biography. Invariably they represent Pythagoras as an outstanding figure even among the great. The common man, so-called, may know almost nothing of science. Yet he seems instinctively to recognize a scientists of absolutely the first rank when, at intervals of centuries, such a one appears. As a memorable instance even the most highly cultured of Newton's contemporaries were incapable of understanding his epochal achievement. Yet somehow they, and the man with no pretensions to culture, knew that here was a scientific mind without a superior in history. And when Einstein appeared, the same instinct for a revolutionary advance in science again awoke, though only one in thousands could follow the mathematical technicalities of relativity. More talent and what in an average century is first-rateness never evoke this instinctive response.

Pedants, intellectual snobs, and worshippers of the second rate may decry this popular recognition of the highest achievement as but one more proof that the public loves a sensation. But for all their envy, they cannot repress the second instinct of their wiser fellows for supreme greatness. And for his part, the man knows little or nothing of science offers the master his tribute of an anecdote, more likely than not without foundation in fact, which epitomizes what the great man has meant to him.

So it was with Pythagoras. Universally recognized as a master among masters, he lived far beyond the narrow confines of his own body, in the wonder and respect of his unlearned fellow men. The legends of his life are not about Pythagoras, they are Pythagoras, and it matters nothing at all whether every last one of them is false or whether it is true. From all the hundreds of doings and sayings attribute to Pythagoras, anyone may accept or reject what he pleases. These legends that accord with a particular individual's conception of the man's essential greatness will be acceptable to that individual. The others may be rejected as the stupid fabrications of dullars incapable of appreciating the master.

Even in antiquity Pythagoras was a dim and legendary figure. Aristotle, for example, born in 384 and dying in 322 B.C., was only about two centuries later than Pythagoras; yet he seems to be in some doubt whether such a human being as Pythagoras ever existed, mentioning him by name twice. Rather than commit himself by referring to the teachings of Pythagoras, Aristotle cautiously attributes the sound mathematics, the music, the harmonious astronomy, and the fantastic numerology, traditionally ascribed to the master himself, to anonymous Pythagoreans. The very name Pythagoras, recalling the Greek for one who is inspired, seems to have been regarded by the over-suspicious as a feeble pun on the Greek (python) for a soothsayer. It followed for the rigidly skeptical that Pythagoras was not a man but a nameless oracle.

Aristotle was justified in being moderately cautious in crediting Pythagoras himself with specific discoveries. For it is almost certain that many of the things branded with the master's name were the inventions of his disciples. Some indeed were made long Pythagoras, either as a man of flesh and blood or as a nebulous hypothesis, had passed on to a stage of existence other than the human. Even during his supposed mortal lifetime Pythagoras was given the credit for all discoveries made by his disciples, somewhat as the director of a scientific research laboratory today occasionally monopolizes the publicity for his entire staff. But, however remote Pythagoras had become by the time biographies of him appeared, the massed testimony of ancient historians establishes his material existence for the majority of critics beyond any reasonable doubt.

The unique contemporary notice of Pythagoras that has survived is a sour tribute to the misanthropic philosopher Heraclitus. This renowned lover of wisdom flourished about 500 B.C. He was nicknamed the Weeping Philosopher, and, appropriately enough, is remembered in histories of philosophy for his too hasty generalization that “All things flow.” Evidently suffering from a twinge of professional jealousy, Heraclitus had this to say of his more successful competitor for immortality; “Pythagoras, son of MNesarchus, has pursued research and inquiry more assiduously than any other man. He has compounded his wisdom from polymathy and bad arts.”

This at any rate preserves the name of Pythagoras earthly father. Mnesarchus was a stonecutter of Samos, where Pythagoras was born at some doubtful date between 580 and 569 B.C. Beyond a vague rumor that Pythagoras’ mother was of Phoenician extraction, little is known of her except that she is said to have accompanied her wandering son of his last journey.

Like some other of the major prophets of our race. Pythagoras to his disciples was divine. His heavenly father was generally believed to have been Apollo. In proof of his celestial descent. Pythagoras when properly approached would display a golden thigh. This curious legend is so persistent that it may be the miracle-mongered distortion of some real physical infirmity.

The acid tribute of Heraclitus merits serious consideration as the unsolicited testimonial from one lover of wisdom to another. "Polymathy" means simply encyclopaedic knowledge, surely no serious disqualification for a man who undertook to sum up the universe in a single formula. But it is obvious that Heraclitus was paying no compliments. By "polymath" he seems to have intended an unpraiseworthy eclecticism, the implication being that Pythagoras lifted whatever he found it and succeeded none too well in assimilating his thefts.

If Heraclitus was right and not just envious, Pythagoras was merely an inferior and rather disreputable magpie. But Heraclitus had the same opportunities as Pythagoras to make something of all the knowledge and wisdom lying about everywhere in that opulent sixth century, and he made little or nothing of what he might have had for the taking. Pythagoras may have appropriated everything he could seize, but he did not stop at that. The rough stones he gathered were gems when he passed them on. And to close the account between these two contestants for a place in human remembrance, it is the fact that for anything of significance contributed by Heraclitus to life as now lived, Pythagoras outranks the disgruntled philosopher in approximately the ratio of infinity to one.

All legends of Pythagoras make him a tireless traveler until he is well into his middle years. It is not recorded at what age he left his native Samos; but it is said that as a youth of eighteen he fell in with Thales. If he did not actually become one of the Wise Man's pupils, Pythagoras absorbed the Thalesian philosophy and mathematics as second hand from Anaximander. He must have been properly impressed; for when Anaximander assured him that the true wisdom was to be mastered only in Mephis, Pythagoras immediately set out for Egypt without a penny in his wallet. A more romantic but less credible legend pictures Thales himself initiating Pythagoras into the mysteries of Zeus on sacred Mount Ida and urging the young man to get to Egypt as fast as he could, even if he had to walk the whole way.

Some accounts credit Pythagoras with all of twenty-two years among the learned men of Egypt and Babylon. Others have been wandering restlessly all over Egypt, Mesopotamia, Phoenicia, India, and even Gaul, far beyond the Pillars of Hercules, and declare moreover that he absorbed all the knowledge and wisdom of the Hebrews, the Persians, the Arabs, and the blue Druids of Britain.

From what is now known of pre-Greek mathematics in Egypt and Babylonia it is not improbable that Pythagoras learned much about numbers and figures from the slowly expiring civilizations of the Near East, regardless of whether he ever lived among their peoples. The number magic that he brought back with him to Samos is almost as valid evidence as travel in the East as a cancelled passport. And though he may never have set foot in India, his missionary zeal for the doctrines of reincarnation and the transmigration of souls is enough to prove that he must have studied under some master thoroughly versed in the religious of the Orient. It does not seem to be definitely known as far west these doctrines had penetrated in the sixth century B.C., nor, for that matter, where they originated. One thing seems certain; they were foreign to the Hellenic genius Pythagoras incorporated them into his own teachings. In a somewhat cruel form they passed thence into the eschatology of Plato, who added the characteristic touch that the souls of cowards are reincarnated in the bodies of women. The souls of the stupid, according to Plato, animate four-footed beast and birds, while an utterly worthless soul, being unworthy to breathe pure air, must content itself with the body of a fish. Pythagoras was more merciful.

It is interesting in reviewing the life of another, if not one's own, to observe the critical points where a slight deviation from the path actually followed might have led to complete success instead of partial failure. Pythagoras passed through one such point when he decided to leave Samos and study in Egypt. Had he not left his fellow Greeks to their accustomed ways while he sought knowledge and wisdom in the East, his name might be as unknown to us as those of the thousands who stayed at home and lived out of their simple lives in ignorance and peace. In all there were three decisive turning points in this fated man's life. The second changed his course directly towards the final catastrophe. At the age of forty (about 530 B.C.), Pythagoras returned to Samos.

His fatal blunder was that which has undone many a prophet. He attempted to raise his own people to his own level. At the height of his enthusiasm for all the mysticism, the mathematics, and the number lore he had acquired, Pythagoras immediately set about the enlightenment of those he had left behind to perpetuate the burden of traditions of their rustic forefathers. In his pathetic simplicity he engaged the municipal amphitheater. Expecting to see it crammed to the sky, the sanguine philosopher found himself staring into a gulf of empty stone benches.

The populace was more than mere indifferent. It was thoroughly roused and enraged. What could Pythagoras, the son of Mnesarchus the stonecutter, know about anything? Why, men of his own age even remembered him as a little boy running about his father’s workshop, getting in everybody’s way, and pestering the customers with stupid questions. He was always asking about the silliest things of no practical use to stonecutters to anyone else. Was it believable that this obnoxiously ignorant little boy could have grown up into a mature man with sound sense in his head? It was not. As an idle adolescent always loafing about the street corners and waylaying the leading citizens for interminable cross-examinations on matters of no importance, this fellow who now called himself a lover of wisdom was even more insufferable than he was as a boy. Let him love wisdom as hotly as he liked; they had their work to do and needed a good night’s sleep after the long day’s toil and sweat. It was just like the conceit to put his proposed lecture after supper when everyone was tired and full of meat.

Then too there was the ridiculous incident of the chastised dog. A man had a right ot beat his dog. It was his, and he could do what he liked with his own property for any good reason or for no reason at all. But this crazy crank Pythagoras had made quite a fuss when he saw one of the prominent citizens taking a stick to his dog. “Stop beating the dog!” he had shouted like a madman. “In his howls of pain I recognize the voice of a friend who died in Memphis twelve years ago. For a sin such as you are committing he is now the dog of a harsh master. By the next turn of the Wheel of Birth, he many be the master and you the dog. May he be more merciful to you than you are to him. Only thus can he escape the Wheel. In the name of Apollo my father, stop, I shall be compelled to lay on you the tenfold curse of the tetractys."

So his father was now Apollo, was he? Since when had that doddering old Mnesarchus, ready to stumble into his grave at any moment now, been one of the immortal gods? This impostor Pythagoras was worse than a nuisance; he has as mad as a goat with a broken head. What right had he to go about scarring healthy people into their beds with his foreigner curses? If the owner of the dog died, they would now what to do with the man who killed him. In the meantime he could continue to lecture to the wind.

There is no record of what Pythagoras said – or thought – of his reception by his own people. Unlike another famous teacher he did not vent his disappointment in petulance. If they would not come to hear him, he would take his message to them. He left the empty amphitheater and got himself a pupil – just one, and poverty-stricken as that. Under the circumstances he might have been pardoned had he said, "Let him which is filthy be filthy still!" But Pythagoras really was a philosopher, and he knew that one of the ways of loving wisdom is to share it with others. What he yearned most ardently to share was his passion for geometry as a deductive science.

Going far beyond Thales, Pythagoras has discovered and proved numerous theorems in what is now the first course in school geometry. Always remembering that some of the theorems attributed to Pythagoras may have been the discoveries of his disciples, we can assert, on the authority of Greek historians of mathematics, that Pythagoras cast geometry in the shape it was to retain for two thousand years. He is credited with having recognized that definitions must be set out in the beginning of the entire subject, and that the postulates (axioms) from which deduction is to proceed must be stated explicitly. Further, he strove in his own proofs to guard against the surreptitious entry of further assumptions in addition to those of the admitted postulates.

It was like a game: here are the pieces, only certain rigidly prescribed rules for moving them are permitted; what are the possible configurations of the pieces in an honestly played game? The pieces are the definitions and postulates; the rules for the moves are those of formal logic; the possible configurations are the deductions from the postulates by means of the logic; and the outcome is the theorem of geometry.

When fully formalized any mathematical argument proceeds by the same tactic: definitions and postulates; deductions; theorems. The rigidity of the Greek technique, but not its underlying logical justification as one deductive science among many, was relaxed in the creation (1637) by Descartes of analytic geometry, in which all the machinery of algebra and mathematical analysis is applied to geometry. The gain in power and simplicity was tremendous, and the strict Greek model passed out of use. But the underlying philosophy remained.

To make it worth his impecunious pupil's time to play the game, Pythagoras paid him a penny for each geometrical theorem he mastered. This suited the poor young man perfectly. By merely sitting in the shade, using his eyes and listening attentively he earned better wages in an hour than he could have made in a full day of back-breaking labor in the broiling sun. But Pythagoras, disciple of the tight-fisted Thales, was not letting the pennies get away from him never to return. Just as his pile began to grow to respectable sum the pupil, in spite of himself, because avidly interested in geometry and begged his teacher to go faster. The irrepressible Greek in Pythagoras saw his chance and reached for the stakes. Confiding to his pupil that he was a desperately poor man himself, Pythagoras suggested that the pupil now to pay him a penny apiece for each new theorem. By the time the young man had absorbed all the geometry he could hold and was ready to return to hard work, Pythagoras had gained back all this money and had exactly as much geometry left as when the game began.

It must be admitted that the last of this story is hardly in character with the master's habitual austere integrity. It may be a late fable invented to dramatize the fact that it is impossible to diminish intangibles by subtracting intangibles from them, or by sharing them with others.

Assured that his pupil was now thoroughly indoctrinated with the new truth, Pythagoras made a second and last effort to enlighten his own people. Always as shrewd in psychology as he was wise in geometry, the master revised his grand strategy. His failure had been his own fault. He should have returned to his native town not as a professor of knowledge and wisdom, but as a pompous mystagogue endorsed by one or more of the leading oracles. Announcing that he was leaving Samos to master the sacred mysteries in Delos and Crete, Pythagoras promised his detractos that he would return as soon as he acquired the proper credentials for instructing them in matters of the highest practical value.

He kept his word. Possibly on his visit to Crete he learned something himself. For there he may have been heard of Epimenides the Cretan. Epimenides the Cretan is justly immortal for his cynical remark, “All Cretans are liars.” Was he lying when he said that? Or was he speaking the truth? Either answer contradicts itself. This was the first of several logical paradoxes that were to perplex the Greek philosophers and mathematicians. If Pythagoras heard that Epimenides had made that disturbing statement he might well have felt uneasy for the security of some of his own. It seemed to slip through the mesh of deductive reasoning like an eel. Was such reasoning as powerful and as sound as the master had thought?

Perhaos it was fortunate for the futures of science and mathematics that Pythagoras either did not hear what Epimenides said or ignored it. Otherwise all the subtle logical difficulties, the “Epimenides” paradox among them, that appeared in the foundations of mathematics toward the close of the nineteenth century A.D., might have prevented Pythagoras from laying the cornerstone of all mathematics in the sixth century B.C.

On his return from ostensibly consulting the oracles, Pythagoras found the Samians a little less friendly. After all they were only human. In spite of their hostility toward their would-be uplifter they began to become curious about him. There was a rumor that he had subdued an enormous bear, that was ravaging the communal pigsties, by simply pointing his finger at the beast and commanding it in the name of Apollo to desist. Then, too, there was all this talk about what he ate, or rather about what he sould not eat. What could the man possibly have against beans? They were a staple of everyone's diet; and here was Pythagoras refusing to touch them because they might harbor the souls of his dead friends. Did anyone ever hear of such nonsense? He had even deterred a cow from trampling a patch of beans by whispering some magic word in it ear. Absurd!

But his ban against eating the flesh of animals might be Worthing looking into. For sho could say that the souls of the dead did not pass into the bodies of animals when there was a shortage of new human bodies of animals when there was a shortage of new human bodies to accommodate all the souls suddenly released in a battle? Pythagoras himself, though not claiming openly to have inhabited the bodies of animals in previous incarnations, insinuated that even he might have done so for his impieties. His decsriptions of some of the lives he had lived in human or divine shape was singularly detailed and eerily convincing. Horrible dreams of their own, when remembered and analyzed in the unearthly light of this sudden new knowledge, hinted that the transmigration of souls might be the dreadful fact Pythagoras said it was. What if it were? The thought of all the souls they might have left shivering in the void by devouring their own goats and swine made the good Samians extremely unhappy. A few weeks more of these upsetting suggestions, and they would all be strict vegetarians – except for beans.

Equally upsetting was the ghastly thought that some of their own children might be malicious little monsters with no souls to restrain their bestial instincts. For Pythagoras had assured them that the total number of souls in the universe is constant. Perhaps he was right in scolding them for having such large families and warning them never, no matter how compelling the urge, to have more than ten children apiece. There was something about the number ten that made an eleventh child repeat all the disagreeable characteristics of the first. They did not understand this very clearly, but it must be true – "Himself said it." For all of a week they conducted their lives as Pythagoras said they should if they were to escape the Wheel of Birth.

Most disturbing of all, that boy Pythagoras had paid to do nothing kept bragging about his mysterious powers as if he were a wizard himself. What in the name of Zeus was a hypotenuse? And how could the square on the hypotenuse of a right-angle triangle be equal to the sum of the squares on the other two sides, if nobody knew what a hypotenuse was? The conceited young upstart told them it didn't matter whether anyone knew; he could "prove" all that rigmarole about the squares. When he proceeded to do so his unreceptive elders gathered that "proof" meant scratching a tangle of lines in the dust with a pointed stick. It looked like some new knid of magic. Probably it was. They decided that it must be magic of a potent kind when the boy told them that Pythagoras had paid Apollo a hecatomb for this magnificent "theorem."

According to the enthusiastic pupil, Pythagoras had actually sacrificed a round hundred of prime cattle to his heavenly father when the latter divulged all the truth about the hypotenuse of any right-angled triangle. The Egyptians and Babylonians had urged Pythagoras to ask Apollo what the truth might be about any hypotenuse. They already knew what it was for a right-angled triangle with two equal sides; and some of them had even suspected the tremendous general truth when Apollo revealed it to his son. What was more, if the boy could be believed, it was Apollo himself who showed Pythagoras how to "prove" this greatest theorem in all "geometry." Now anybody with just a little sense could do the same. It was easy when he knew how. Possibly; but the elders doubted. In any event they were not going to have their bright boys turned into haughty young wizards under their noses. This sort of thing must be stopped.

It was. The conservative element laid the facts in the case before the tyrant of Samos, their good friend and dictator of their minds. This able despot was shrewd enough to know that the only competitor he need fear was brains. Obviously this man Pythagoras was intelligent to a fault. The tyrant invited him to leave Samos.

At this critical juncture in his career Pythagoras hesitated. Should he submit to the tyrant and desert his own people? Or should he stay and share with them as much of his hard-gained enlightenment as they were capable of receiving? He knew that the tyrant, a petty opportunist with the mediocre mind of a practical politician, would be no match for him in a contest for the people's loyalty. He could win them to his side in a week, if he so willed; and they would hound their tyrant over a cliff into the sea. In a pinch he could cow them with a trick or two of the childish magic he had learned from the

Egyptian priests. That old one of changing a serpent into a rod, and back into a serpent, alone would be enough to make the people his slaves. Should he go, or should he stay? Plainly his earthly father had not much longer to live. Soon his ageing mother would be the only personal tie binding him to Samos. Not quite the only one: there was this brilliant boy, his first disciple, who must be given his chance to become a geometer. What ever the decision, his one convert to reason must not be abandoned.

The appreciate the choice Pythagoras made, we may contract his background in the sic=xth century B.C. against our own. It has been said that the white race is divided into two irreconcilable factions: those who regret that the French Revolution of 1789 ever started; those who regret that this democratic upheaval stopped before it had well started.

In the sixth century B.C. there were no machines to lighten the heavy work. Consequently slavery for the majority was a necessity if a minority was to live comfortably and have leisure, among other things, to think. Democracy as the French revolutionists imagined it might become, was not even a philosopher's dream in the time of Pythagoras. There was demos, the mob, from which our word democracy is derived, and there was aristos, the best, from which we get our word aristocracy. Democracy means, literally, rule by the mob, aristocracy, rule by the best.

The slaves were not counted among the best in the sixth century B.C. any more than are all our brainless machines included in our aristocracy, though it may be true that they rule us. It was natural for Pythagoras to ignore ninetyfive – or more – out of every hundred of his fellow Samians in reaching his decision as it would be for us, in a similar crisis, to make our minds without consulting our machines. Slaves and machines, the one at the highest subhuman to the philosophic mind, the other without souls of any kind to the scientific eye, would be taken for granted as commonplace necessities for the good life.

The best of his own people had rejected Pythagoras. Though their fear was an unwitting tribute to respect, they probably were beyond lasting enlightenment. But he had recently heard of a Doric colony at Croton in Southern Italy, governed by true Greek aristocrats. They would welcome the new wisdom. The light Pythagoras hoped to kindle in Croton would illuminate the whole world.

In making his decision Pythagoras had in mind both science, including mathematics, and his secret number magic. These were to be the new light of the world. The farthest-reaching consequence of his choice he could not possibly have foreseen. Nor would he have understood if it had been shown to him. The slavery which made it possible for him to live and think, and which he had barely noticed, was to be ameliorated, if not abolished, by the natural growth of that part of all his knowledge and wisdom which he deemed the lesser. What he considered the greater part was to obstruct enlightenment and foster superstition.

Accompanied only by his mother and his disciple, Pythagoras sailed from Samos. He had made the third and last slight turn in his course. His path now lay straight before him to his glory and his doom.

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Heron’s Theorem about the Area of a Triangle

Heron’s Theorem about the area of a triangle dates back to the first century A.D. Heron (10 to 85 A.D.) lived and worked at the great University and Library in Alexandria. He is most famous for his Theorem which relates to the area of a triangle of any shape. A simple derivation of Heron’s Theorem is provided. This theorem and its derivation should be known by all high school mathematics students.

Calculating the Area of a Triangle using its Three Sides.

Consider a triangle with sides a, b and c (Fig. 1). The height of the triangle is h. Let us divide side a, into m and n, such that m + n = a.

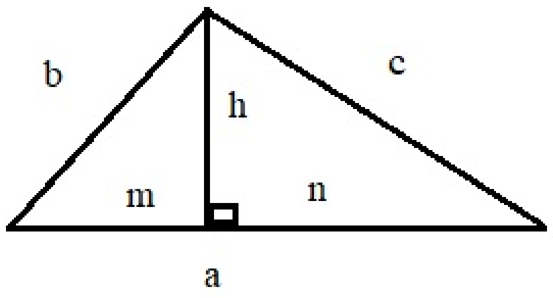

Fig. 1: The sides a, b, c and height h of a Triangle

Using Pythagoras’ Theorem for each of the two smaller triangles we find that

$$m^2 + h^2 = b^2$$

$$n^2 + h^2 = c$$

Subtract the second equation from the first we get

$$m^2 - n^2 = b^2 - c^2 = \ (m + n\ )(m - n) = a(m - n)$$

From this we find that

$$(m - n) = \ \frac{b^2 - c^2}{a}$$

Solving for m and n separately we find

$$m = \frac{a^2 + b^2 - c^2}{2a}$$

$$n = \frac{a^2 - b^2 + c^2}{2a}$$

Returning to the height h we find that

$$h^2 = \ b^2 - m^2 = b^2 - \left(\frac{a^2 + b^2 - c^2}{2a}\right)^2$$

Expanding this out and gathering similar terms we find that

$$h^2 = \ \frac{[(a + b)^2 - c^2][c^2 - (a + b)^2]}{4a^2}$$

Expanding out the terms we get

$$h^2 = \frac{(a+b+c)(a+b-c)(c+a-b)(c-a+b)}{4a^2}$$

In a stroke of genius Heron introduces the *semi-perimeter measure* s defined as

$$s = \frac{(a+b+c)}{2}$$

In which case the terms become

$$(a+b-c) = 2\ (s-c)$$

$$(a-b+c) = 2\ (s-b)$$

$$(c+b-a) = 2\ (s-a)$$

which means that

$$h^2 = \frac{(2s)2\ (s-c)2\ (s-b)2\ (s-a)}{4a^2}$$

By inspection we find that the area of the triangle is given by

$$A = \frac{a\ h}{2} = \sqrt{s\ (s-a)(s-b)(s-c)}$$

which is Heron’s Theorem for calculating the area of a triangle with sides a, b & c.

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